1-6 Practice

Function Operations and Composition of Functions

Find \( f + g \), \( f - g \), \( f \cdot g \), and \( \frac{f}{g} \) for each \( f(x) \) and \( g(x) \). State the domain of each new function.

1. \( f(x) = 2x^2 + 8 \) and \( g(x) = 5x - 6 \)
   \( f(x) + g(x) = 2x^2 + 8 + 5x - 6 = 2x^2 + 5x + 2, D = (-\infty, \infty) \)
   \( f(x) - g(x) = 2x^2 + 8 - 5x + 6 = 2x^2 - 5x + 14, D = (-\infty, \infty) \)

2. \( f(x) = x^2 \) and \( g(x) = \sqrt{x + 1} \)
   \( f(x) + g(x) = x^2 + \sqrt{x + 1}, D = [-1, \infty) \)
   \( f(x) - g(x) = x^2 - \sqrt{x + 1}, D = [-1, \infty) \)

3. \( f(x) = 10x^2 - 12x^4 + 40x - 48 \), \( g(x) = x^2 \sqrt{x + 1} \)
   \( f(x) + g(x) = 10x^2 - 12x^4 + 40x - 48 + x^2 \sqrt{x + 1}, D = [-1, \infty) \)
   \( f(x) - g(x) = 10x^2 - 12x^4 + 40x - 48 - x^2 \sqrt{x + 1}, D = (-1, \infty) \)

4. \( f(x) = 2x^3 + 8 \) and \( g(x) = 5x - 6 \)
   \( f(x) \cdot g(x) = (2x^3 + 8)(5x - 6) = 10x^4 - 12x^3 + 40x - 48, D = \{x \neq \frac{6}{5}, x \in \mathbb{R}\} \)
   \( \frac{f(x)}{g(x)} = \frac{2x^3 + 8}{5x - 6}, D = (-1, \infty) \)

For each pair of functions, find \( f \circ g \) and \( g \circ f \).

5. \( f(x) = x + 5 \) and \( g(x) = x - 3 \)
   \( f(g(x)) = x + 5, g(f(x)) = x - 3 \)

6. \( f(x) = x^2 - 2x + 4 \)
   \( g(x) = x^2 + 5 \)
   \( f(g(x)) = (x^2 + 5)^2 - 2(x^2 + 5) + 4 = x^4 + 8x^2 + 16, D = \mathbb{R} \)
   \( g(f(x)) = (x^2 - 2x + 4)^2 + 5 = x^4 - 4x^3 + 13x^2 - 20x + 19, D = \mathbb{R} \)

7. \( f(x) = \sqrt{x - 2} \), \( g(x) = x^2 + 5 \)
   \( f(g(x)) = \sqrt{x^2 + 5 - 2} = \sqrt{x^2 + 3}, \{x \in \mathbb{R}, x \geq 0\} \)
   \( g(f(x)) = (\sqrt{x - 2})^2 + 5 = x - 2 + 5 = x + 3, \{x \in \mathbb{R}, x \geq 2\} \)

8. \( f(x) = -\frac{1}{x}, g(x) = \frac{1}{x} \)
   \( f(g(x)) = f(\frac{1}{x}) = \frac{-1}{\frac{1}{x}} = -x, \{x \neq 0\} \)
   \( g(f(x)) = g(-x) = \frac{1}{-x}, \{x \neq 0\} \)

9. \( h(x) = \sqrt{x - 6} - 1 \)
   \( h(3) = \sqrt{3 - 6} - 1 = -2, D = [6, \infty) \)

10. \( h(x) = \frac{1}{3x + 3} \)
    \( h(2) = \frac{1}{3(2) + 3} = \frac{1}{9}, D = \{x \in \mathbb{R}, x \neq -1\} \)

11. RESTAURANT A group of three restaurant patrons order the same meal and drink and leave an 18% tip. Determine functions that represent the cost of all of the meals before tip, the actual tip, and the composition of the two functions that gives the cost for all of the meals including tip.
    \( f(x) = 3x \), where \( x \) is the cost for one meal; \( g(x) = 1.18x \); \( g(f(x)) = 3.54x \)

1-6 Word Problem Practice

Function Operations and Composition of Functions

1. MARCHING BAND Band members form a circle of radius \( r \) when the music starts. They march outward as they play. The function \( f(t) = 25t \) gives the radius of the circle in feet after \( t \) seconds. Using \( \pi \cdot r^2 \) for the area of the circle, write a composite function that gives the area of the circle after \( t \) seconds. Then find the area, to the nearest tenth, after 4 seconds.

   \( g(t) = 6.25\pi t^2; 314.2 ft^2 \)

2. CANDLES A hobbyist makes and sells candles at a local market. The function \( c(h) = 46 \) gives the number of candles she has made after \( h \) hours. The function \( f(c) = 12 + 0.25c \) gives the cost of making \( c \) candles.
   a. Write the composite function that gives the cost of candle making after \( h \) hours.
   \( f(c(h)) = 12 + h \)
   b. A sale reduces the cost of making \( c \) candles by 10%. Write the sale function \( s(c) \) and the composite function that gives the cost of candle making after \( h \) hours if materials are purchased during the sale.
   \( s(c) = 0.9c; s(f(c(h))) = 10.8 + 0.9h \)

3. SCIENCE The function \( f(x) = t(x) = \sqrt{2x} + 6.25 \) gives the temperature in degrees Celsius of the liquid in a beaker after \( x \) seconds. Decompose the function into two separate functions, \( a(x) \) and \( b(x) \), so that \( \{a + b\}(x) = f(x) \) and \( a(x) \) is a quadratic function and \( b(x) \) is a linear function.
   Sample answer: \( a(x) = 2x^2; b(x) = x - 3 \)

4. TRAVEL Two travelers are budgeting money for the same trip. The first traveler's budget (in dollars) can be represented by \( f(x) = 45x + 350 \). The second traveler's budget (in dollars) can be represented by \( g(x) = 60x + 475 \) and the number of nights.
   a. Find \( f + g \) and the relevant domain. \( f + g(x) = 105x + 825; D = \{x \in \mathbb{R}, x \in \mathbb{Z}\} \)
   b. What does the composite function in part a represent? the combined budget of both travelers
   c. Find \( f + g \) and explain what the value represents. \$1560; the combined amount that can be spent by the travelers on a 7-night trip
   d. Repeat parts a-c for \( g - f \).
      a: \( g - f(x) = 15x + 125; D = \{x \in \mathbb{R}, x \in \mathbb{Z}\} \)
      b: how much more the second traveler can spend than the first
      c: \$230; how much more the second traveler can spend on a 7-night trip

5. POPULATION The function \( p(x) = 2x^2 - 12x + 18 \) predicts the population of elk in a forest for the years 2010 through 2015 where \( x \) is the number of years since 2000. Decompose the function into two separate functions, \( a(x) \) and \( b(x) \), so that \( \{a + b\}(x) = p(x) \) and \( a(x) \) is a quadratic function and \( b(x) \) is a linear function.
   Sample answer: \( a(x) = 2x^2; b(x) = x - 3 \)